

# Analysis and Design of Simply Supported Deep Beams Using Strut and Tie Method

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**Abstract:** Generally structural members can be broadly divided into two regions, namely B or Bernoulli regions where the strain distributions are linear and D or Disturbed regions where the strain distributions are nonlinear. A beam whose depth is comparable to span is known as deep beam and these structural elements belong to D regions. It has been recently understood that the strut and tie method (STM) is an effective tool for the design of both B and D regions. The present code recommendations are inadequate for the design of deep beams. In this paper simple equations using STM are developed for finding the area of main steel required to have a balanced type of failure and to find the ultimate capacity of deep beams failing in different failure modes. These equations are compared with experimental results and a good agreement is found.

**Key words:** deep beam, reinforced concrete, strut and tie method, flexure, shear.

## 1. INTRODUCTION

Concrete structural members having depth comparable to span are generally termed as deep beams. In these members, the distribution of strains across depth of the cross section will be nonlinear and the significant amount of load is carried to the supports by a compression strut joining the load and the reaction. These structural elements belong to D (disturbed) regions, which have traditionally been designed using empirical formulae or using past experience. Strut and tie method (STM) offers an alternative to such empirical method. Also STM provides design engineers with a more flexible and intuitive option for designing structural elements. Since STM is a realistic approach, this has found place in many codes like American code (ACI 318-08. (2008)), Australian code (AS 3600. 2001), Canadian

code (A23.3-04), Eurocode (EUROCODE 2. 2004), Model code (CEB-FIP Model Code 1990), New Zealand code (NZS 3101. 2006) etc.

The American code ACI 318-08 (2008) does not contain any recommendations for designing deep beams for flexure and it recommends to either use a non linear analysis or STM for designing deep beams. Many codes have adopted the recommendations given in CEB (1970) which is based on the experimental investigations conducted by Leonhardt and Walther at University of Stuttgart (SP: 24. 1983). For example IS 456 (2000) recommends this procedure for the design of deep beams and it is seen that these recommendations are inadequate for the design of deep beams (discussed later).

Considering the above, an attempt has been made to develop simple equations using STM for the analysis and design of simply supported deep beams.

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## 2. IS 456: 2000 CODE PROVISIONS FOR THE DESIGN OF SIMPLY SUPPORTED DEEP BEAMS

The existing IS 456 (2000) code recommendations are valid only for deep beams subject to uniformly distributed load (UDL). As per the code, when the ratio of the effective span ( $L$ ) to the overall depth ( $D$ ) of a simply supported beam is less than or equal to 2.0, then the beam can be treated as a deep beam. The lever arm ( $Z$ ) is given as

$$\begin{aligned} Z &= 0.2 (L + 2 D); 1 \leq \frac{L}{D} \leq 2 \\ &= 0.6 L; \frac{L}{D} < 1 \end{aligned} \quad (1)$$

where,  $L$  is the effective span taken as centre to centre distance between the supports or 1.15 times the clear span, whichever is smaller and  $D$  is the overall depth. The tensile reinforcement  $A_{st}$  required to resist the positive bending moment can be calculated using the expression

$$\begin{aligned} M_u &= \frac{W_u L^2}{8} = TZ = \frac{f_y}{\gamma_s} A_{st} Z = 0.87 f_y A_{st} Z \\ A_{st} &= \frac{W_u L^2}{0.87 f_y \times 8Z} \end{aligned} \quad (2)$$

where  $M_u$  is the factored bending moment,  $W_u$  is the factored UDL applied on the beam,  $T$  is the tension force and  $f_y$  is the yield stress of the steel used.  $\gamma_s$  is the partial (material) safety factor for steel and which is equal to 1.15 as per IS 456 (2000) code recommendations.

It is seen that the code recommendation is valid for deep beams that fails as under-reinforced beams (beam fails in flexure). In order to use the code recommendations, it is necessary to assume a value for the depth of the beam and further it is necessary to have knowledge of the load acting on the beam. A beam with a given dimension cannot carry a load beyond a certain limit (limiting shear capacity). But the code does not recommend any method to find the limiting capacity of deep beams. Further a beam will fail in flexure only when the area of tensile steel is less than area of steel required to induce a balanced type of failure and there is no code recommendation to find the area of steel for balanced type of failure. Further the code recommendations are valid only for deep beams subject to UDL. However, in actual practice, we may come across in addition to UDL, concentrated loads, trapezoidal loads, triangular loads etc. The present methods do not consider these types of

loading in the design. Hence a general method which takes into account different type of loading is always preferred. Thus it can be seen that the present IS 456 code recommendation is inadequate for the design of deep beams.

## 3. A BRIEF REVIEW OF STRUT AND TIE METHOD (STM)

In STM, a reinforced concrete member is idealized by an equivalent truss, and analysed for applied loads. The compression and tension zones are converted into equivalent struts and ties respectively, which are in turn connected at the nodes to form a statically admissible truss. The STM is based on the lower bound theorem of plasticity. Therefore, the actual capacity of the structure is considered to be equal to or greater than that of the idealized truss i.e. STM underestimates the strength of the reinforced concrete member. Hence, designs based on this method will be always on the safer side. This method is generally used for the analysis, design and detailing of D-regions such as vicinities of point loads, corner of frames, corbels and also where sudden changes in cross-section occurs. Various components in a strut and tie model for reinforced concrete elements are struts, ties and nodes. Struts are compression members in a strut and tie model. The different types of struts are shown in Figure 1. Ties are the tension members in a strut and tie model and they represent reinforcing steel. Nodes form at points where struts and ties intersect. Nodes are described by the type of the members that intersect at the nodes. For example, a CCT node is one which is bounded by two struts (C) and one tie (T). Using this nomenclature nodes are classified as CCC, CCT, CTT or TTT (Figure 2). C is used to denote the compression force and T is used to denote the tension force. For more details regarding STM, Schlaich *et al.* (1987) and SP-208 (2003) can be referred to. A review of various design criteria for STM recommended by different codes of practice can be obtained from Su and Chandler (2001).

In the case of a real truss, the identification of member areas and joint details and their design is fairly straight forward. However, in the case of an implicit truss embedded in concrete, the determination of appropriate member cross sectional areas and node dimensions is not so simple, especially for the determination of the concrete strut and node dimensions. Although IS 456 (2000) recommends the use of the strut and tie method (for corbel design), no guidelines are given for the determination of the dimensions of the struts and nodes and for the permissible stresses in these elements. Hence, the design recommendations given in ACI 318-08 (2008)

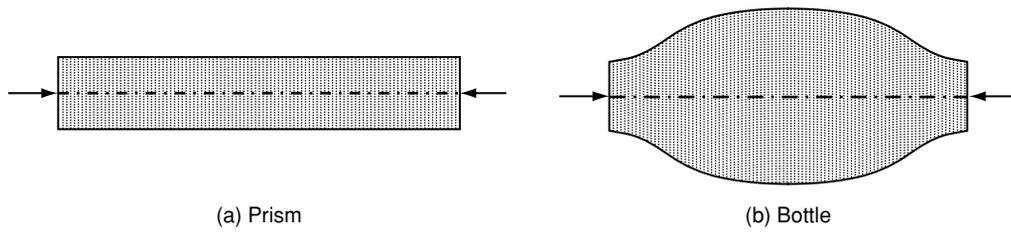


Figure 1. Different types of struts

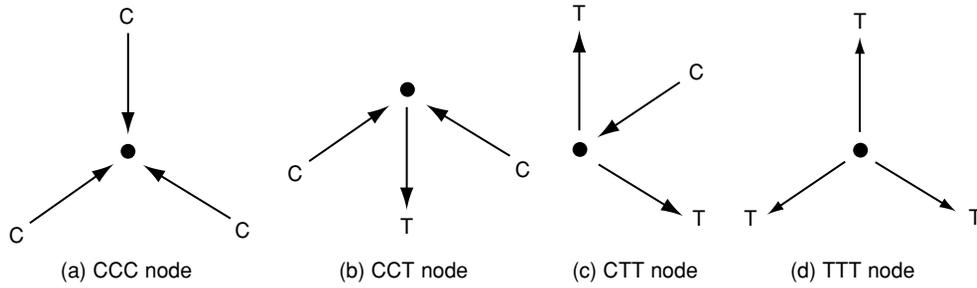


Figure 2. Different types of nodes

are used in this paper and the salient details are given below. The recommendations are slightly modified by incorporating the safety factors and notations followed in IS 456 (2000).

### 3.1. Permissible Stresses in Struts and Nodes

The permissible stresses in different types of struts ( $f_{cs}$ ) is given as

$$f_{cs} = f_{cd} \beta_s \tag{3}$$

where  $f_{cd}$  is the design compressive strength of concrete which is given as

$$f_{cd} = \frac{0.85 f'_c}{\gamma_c} = 0.45 f_{ck} \tag{4}$$

where  $f'_c$  is the compressive strength of concrete cylinder and  $\gamma_c$  is the partial (material) safety factor for concrete and which is equal to 1.5 as per IS 456 (2000) code recommendations. The coefficient 0.85 accounts for the sustained loading.  $f_{ck}$  is the characteristic compressive strength of concrete cube of size 150 mm ( $f_{ck} \approx 1.25 f'_c$ ) and  $\beta_s$  is a stress reduction factor to account for the different types of struts. The values of  $\beta_s$  as per ACI 318-08 (2008) are given in Table 1.

The permissible stresses in different types of nodes ( $f_{cn}$ ) is given as

$$f_{cn} = 0.45 f_{ck} \beta_n \tag{5}$$

where  $\beta_n$  is a stress reduction factor to account for the different types of nodes and its values as per ACI 318-08 (2008) are given in Table 2.

Table 1.  $\beta_s$  for different types of struts

Type of strut	$\beta_s$
Prismatic	1
Bottle shaped (with crack control reinforcement)	0.75
Bottle shaped (with no crack control reinforcement)	0.6

Table 2.  $\beta_n$  for different types of nodes

Type of node	$\beta_n$
CCC	1
CCT	0.8
CTT, TTT	0.6

## 4. STRUT AND TIE MODEL FOR A SIMPLY SUPPORTED DEEP BEAM

Figure 3 shows a simply supported deep beam subject to an arbitrarily distributed load where  $L$  is the effective span,  $D$  is the depth and  $b$  is the width of the beam.

The load distribution that is applied at the top of the beam is resisted by two support reactions  $R_A$  and  $R_B$ . To draw the strut and tie model, the load is subdivided in such a way that the associated resulting loads in the upper part of the structure find their equivalent counterpart on the opposite side (lower part).

The strut and tie model showing the struts width, nodal zone and width of tie for the deep beam is shown in Figure 4. The load distribution shown in Figure 3 is

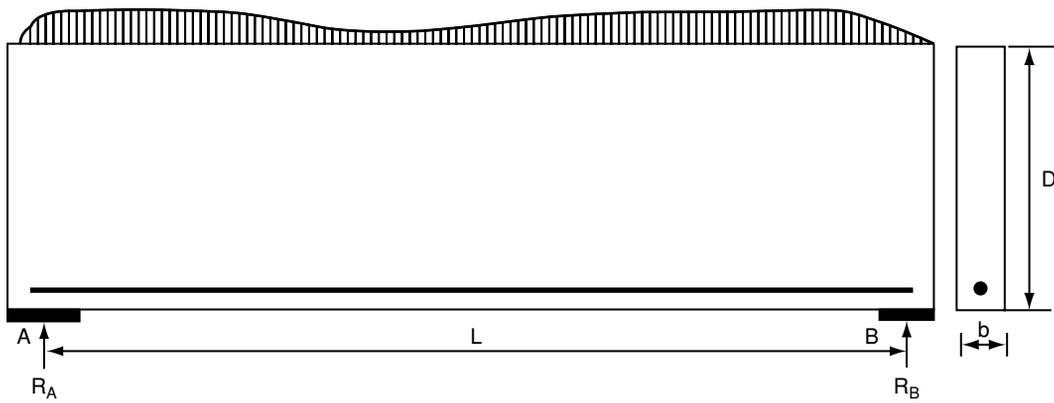


Figure 3. Simply supported deep beam subject to arbitrarily distributed load

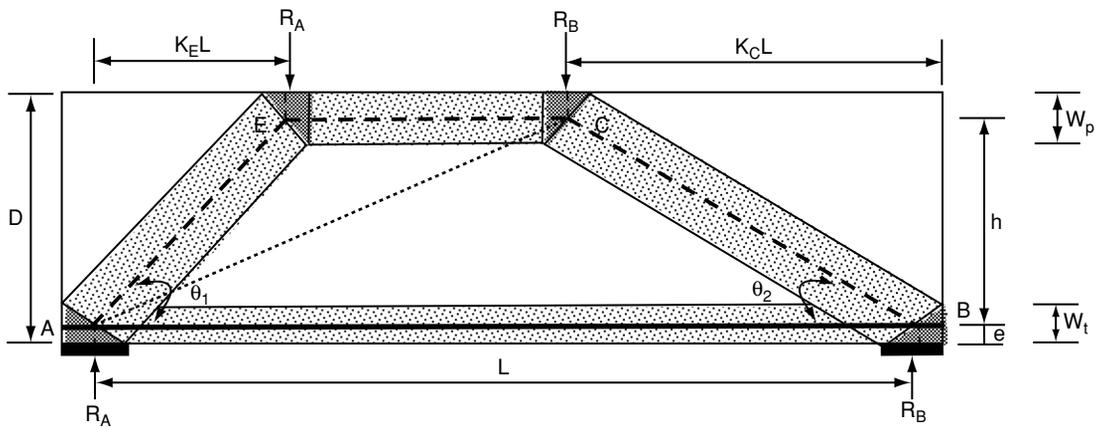


Figure 4. Strut and tie model for a simply supported deep beam subject to arbitrary distribution of load

replaced by equivalent loads  $R_A$  and  $R_B$  as shown in Figure 4. In the figure, struts are shown by dotted lines (since they are not real members) and ties are shown by solid lines. Member AC is included so that the truss is stable. Since the shear force is zero in between E and C, the force in member AC is zero.  $k_E$  and  $k_C$  are the shear span coefficient for the loads at E and C.  $w_p$  is the width of the prismatic strut EC and  $w_t$  is the width of the tie AB which is equal to twice the effective cover  $e$ .  $e$  is the distance measured from the exposed concrete surface to the centroid of the reinforcing bars.

In Figure 4,  $h$  denotes the height of the truss. The height of the truss can be determined by equating the capacities of the prismatic strut EC and the tie AB and assuming that both of the members reaches their limiting capacities. If  $C_P$  is the compressive force in the prismatic strut EC and  $T$  is the force in the tie AB, then

$$C_P = 0.45 f_{ck} \beta_s w_p b \tag{6}$$

$$T = 0.45 f_{ck} \beta_n w_t b \tag{7}$$

$\beta_s$  for a prismatic strut is 1.0 and  $\beta_n$  for a CCT node is 0.8. Equating  $C_P$  and  $T$  we get,

$$w_t = 0.8 w_p \tag{8}$$

Then the height of the truss is given as

$$h = D - \frac{w_p}{2} - \frac{w_t}{2} = D - 0.9 w_t \tag{9}$$

The strut inclination  $\theta_i$  is given by the relation

$$\tan \theta_i = \frac{h}{k_i L} \tag{10}$$

### 5. DETERMINATION OF AREA OF MAIN TENSION STEEL FOR BALANCED TYPE OF FAILURE

The area of steel for balanced type of failure ( $A_{st,b}$ ) can be obtained by assuming the strut AE or BC (failure of inclined strut indicates shear failure) and the tie AB (failure of tie indicates flexure type of failure) reaching their limiting capacities simultaneously. Consider the free body diagram of the node A or B (Figure 5).

In Figure 5,  $R$  is the support reaction,  $C$  is the compressive force in the bottle shaped strut of width  $w_s$ ,

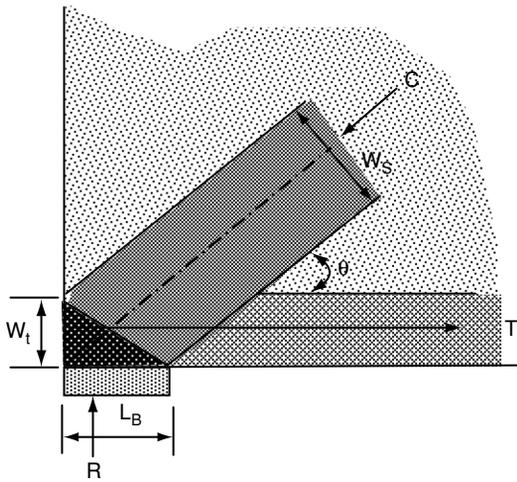


Figure 5. Free body diagram of the node at support

$\theta$  is the strut inclination and  $L_B$  is the length of the bearing plate. From this figure, the strut width can be evaluated as:

$$w_s = w_t \cos\theta + L_B \sin\theta \quad (11)$$

The capacity of the strut and tie is given as:

$$C = 0.45 f_{ck} \beta_s w_s b \quad (12)$$

$$T = 0.87 f_y A_{st,b} \quad (13)$$

From Figure 5, using the equation of equilibrium, we get the relation:

$$C \cos\theta = T \quad (14)$$

By substituting the values for  $C$  and  $T$ , we get:

$$0.45 f_{ck} \beta_s w_s b \cos\theta = 0.87 f_y A_{st,b} \quad (15)$$

Hence

$$A_{st,b} = \frac{0.45 f_{ck} \beta_s w_s b \cos\theta}{0.87 f_y} \quad (16)$$

The corresponding percentage of steel ( $p_{t,b}$ ) is given as:

$$p_{t,b} = 100 \frac{A_{st,b}}{bd} = \frac{51.72 f_{ck} \beta_s w_s \cos\theta}{f_y d} \quad (17)$$

where  $d$  is the effective depth of the beam ( $d = D - e$ )

## 6. PREDICTING THE LOAD CARRYING CAPACITY OF SIMPLY SUPPORTED DEEP BEAMS

If the area of main tension steel ( $A_{st}$ ) provided is less than area of steel for balanced type of failure ( $A_{st,b}$ ), then tie will fail before the bottle shaped strut reaches its limiting capacity and this type of failure can be considered as flexure failure. If  $A_{st}$  greater  $A_{st,b}$ , then the strut will fail

before the tie yields and this type of failure can be considered as shear type of failure. The capacity of the beam for these two failures modes can be determined as discussed below. It is assumed that the anchorage failure is prevented by proper detailing.

### 6.1. Load Carrying Capacity for Simply Supported Deep Beam Failing by Flexure

Since  $A_{st}$  is less than  $A_{st,b}$ , the tie will yield before the strut fails. Hence

$$T = 0.87 f_y A_{st} \quad (18)$$

From Figure 5, the support reaction  $R$  can be found out as:

$$R = T \tan\theta = 0.87 f_y A_{st} \tan\theta \quad (19)$$

By adding the reactions at the two supports, the capacity of the deep beam failing in flexure can be found out. For example, for the deep beam shown in Figure 3, the capacity of the beam in flexure ( $P_F$ ) is given as:

$$\begin{aligned} P_F &= R_A + R_B = T \tan\theta_1 + T \tan\theta_2 \\ &= 0.87 f_y A_{st} (\tan\theta_1 + \tan\theta_2) \end{aligned} \quad (20)$$

### 6.2. Limiting Capacity for Simply Supported Deep Beam Failing in Shear

Since  $A_{st}$  is greater than  $A_{st,b}$ , the inclined struts will fail before the tie fails. Hence

$$C = 0.45 f_{ck} \beta_s w_s b \quad (21)$$

From Figure 5, the support reaction  $R$  can be found out as:

$$R = C \sin\theta = 0.45 f_{ck} \beta_s w_s b \sin\theta \quad (22)$$

The maximum value of the reaction can be taken as the limiting shear capacity of the beam ( $P_S$ ).

## 7. VALIDATION OF PROPOSED METHOD WITH AVAILABLE EXPERIMENTAL RESULTS

The equations developed to predict the balanced area of steel and to predict load carrying capacity of the deep beams was validated using available experimental results (Varghese and Krishnamoorthy 1966; Ramakrishnan and Ananthanarayana 1968; Kong *et al.* 1970; Smith and Vantsiotis 1982; Ray 1984; Rogowsky *et al.* 1986; Selvam and Thomas 1987; Selvam and Harikumar 1990; Tan *et al.* 1995, 1997a, b, 1999; Oh and Shin 2001). For this purpose, 237 deep beam specimens were considered. The area of steel provided in these specimens ( $A_{st}$ ) was first compared with the area of steel required for balanced type of failure and hence the mode of failure was predicted. It was seen that, the mode of failure predicted matches with the failure mode reported in the literature

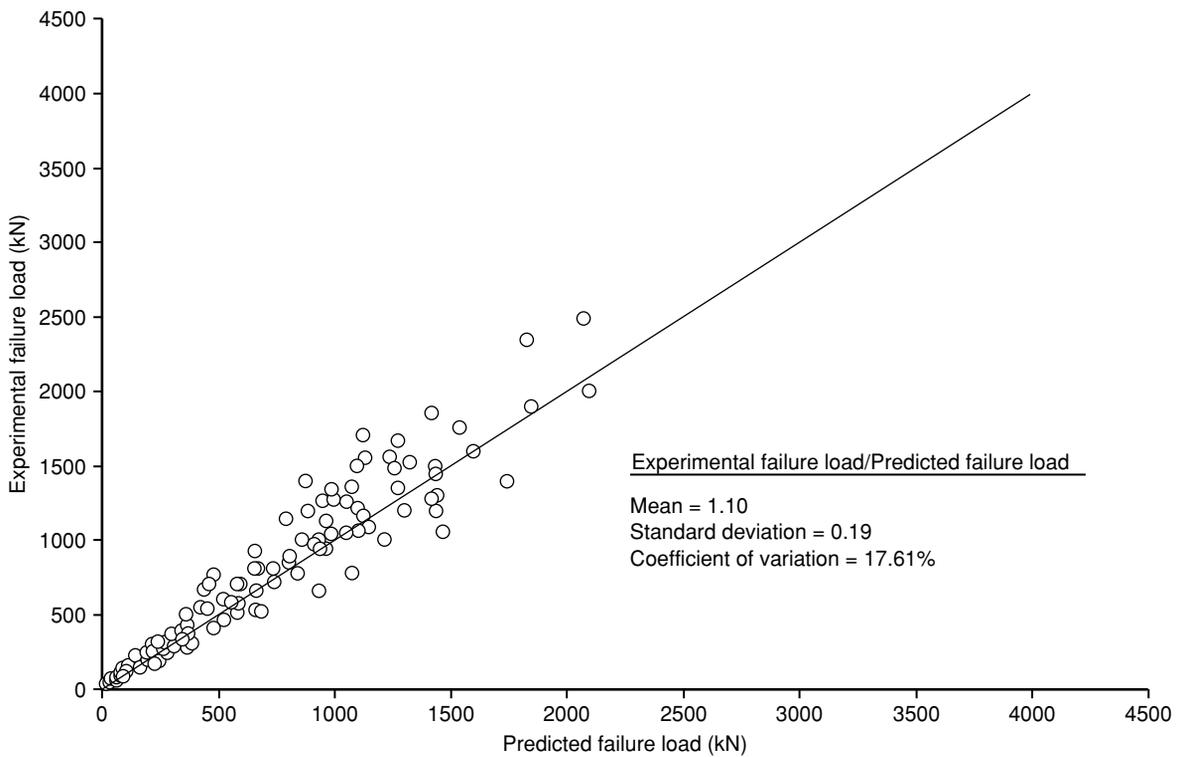


Figure 6. Comparison of predicted failure load with the available experimental results

Table 3. Specimen details and test results

No.	Specimen**	$f_{ck}$ (MPa)	$f_y$ (MPa)	$p_{t,b}$ (%)	Failure load, $P_E$ (kN)	Predicted failure load P(kN)	$P_E/P$	Type of failure	Predicted failure mode
1	UF-0.14/1	27.11	435	0.37	65	52.24	1.24	Flexure-shear	Flexure
2	UF-0.14/2	28	435	0.36	70	52.24	1.33	Flexure-shear	Flexure
3	US-0.57/1	27.11	425	0.37	155	133.57	1.16	Shear	Shear
4	US-0.57/2	28	425	0.38	160	137.96	1.15	Shear	Shear
5	TF-0.14/1	26.67	435	0.44	45	40.6	1.10	Flexure-shear	Flexure
6	TF-0.14/2	27.11	435	0.45	50	40.6	1.23	Flexure-shear	Flexure
7	TF-0.14/3	28.88	435	0.48	60	40.6	1.47	Flexure-shear	Flexure
8	TF-0.25/1	27.55	430	0.47	80	71.66	1.11	Flexure-shear	Flexure
9	TF-0.25/2	29.33	430	0.50	85	71.66	1.18	Flexure-shear	Flexure
10	TS-0.58/1	27.55	425	0.48	145	131.26	1.10	Shear	Shear
11	TS-0.58/2	28	425	0.47	150	133.41	1.12	Shear	Shear
12	CF-0.25/1	27.55	430	0.61	60	47.77	1.25	Flexure	Flexure
13	CF-0.25/2	28.44	430	0.63	65	47.77	1.36	Flexure	Flexure
14	CF-0.40/1	26.67	428	0.61	80	74.18	1.07	Flexure	Flexure
15	CF-0.40/2	27.11	428	0.60	85	74.18	1.14	Flexure	Flexure
16	CF-0.40/3	30.67	428	0.69	90	74.18	1.21	Flexure	Flexure
17	CF-0.51/1	27.55	430	0.61	125	95.55	1.30	Flexure-shear	Flexure
18	CS-0.80/1	26.67	425	0.62	125	111.39	1.12	Shear	Shear
19	CS-0.80/2	27.55	425	0.60	135	115.07	1.17	Shear	Shear
							Mean	1.21	
							Standard deviation	0.10	
							Coefficient of variation	8.63%	

\*\*The first letter stands for type of load; U = UDL, T = Two point load, C = central point load. The second letter stands for type of predicted mode of failure; F = flexure failure, S = shear failure. The numerical value stands for percentage of steel provided ( $p_t$ ) and number of specimens. For e.g. in UF-0.14/1, U stands for UDL, F stands for flexure failure, 0.14 stands for percentage of steel provided and 1 stands for first specimen in each trial.

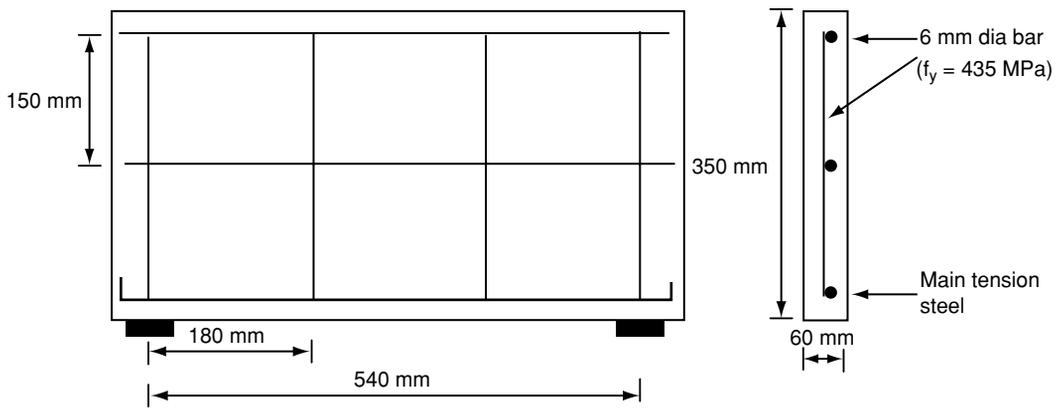
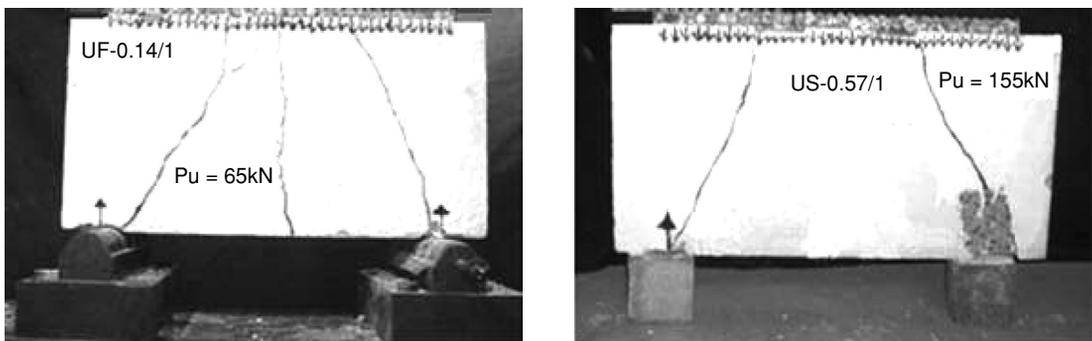
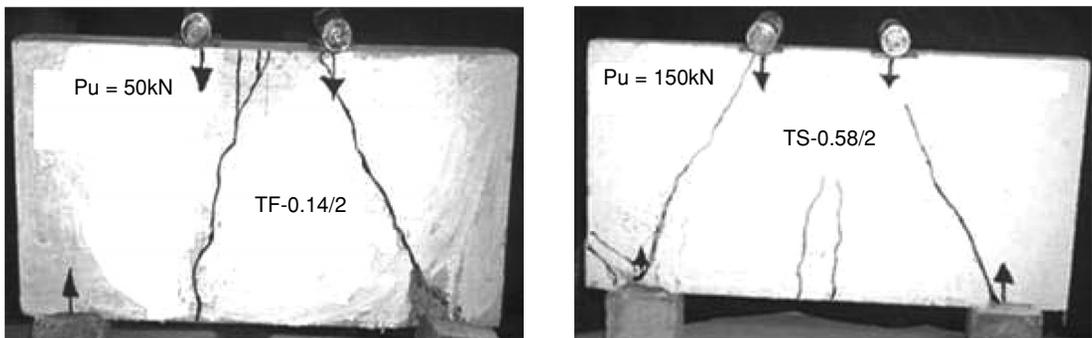


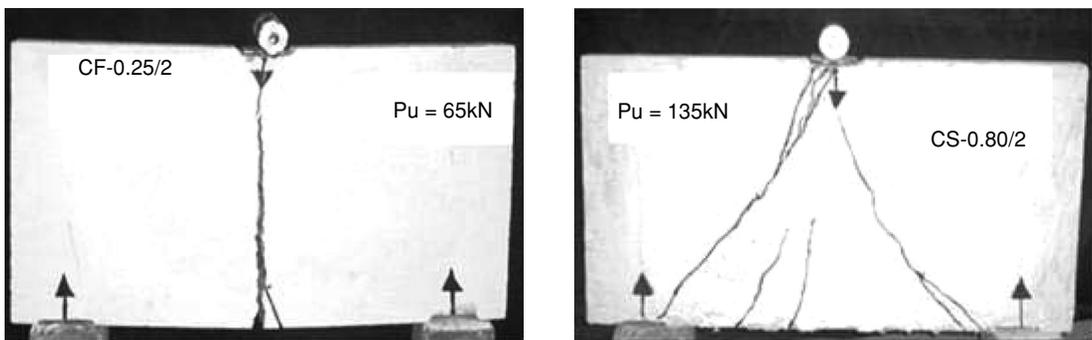
Figure 7. Reinforcement details of the deep beam



(a) Beams tested under UDL



(b) Beams tested under two point load



(c) Beams tested under central point load

Figure 8. Failure modes of some of the beams

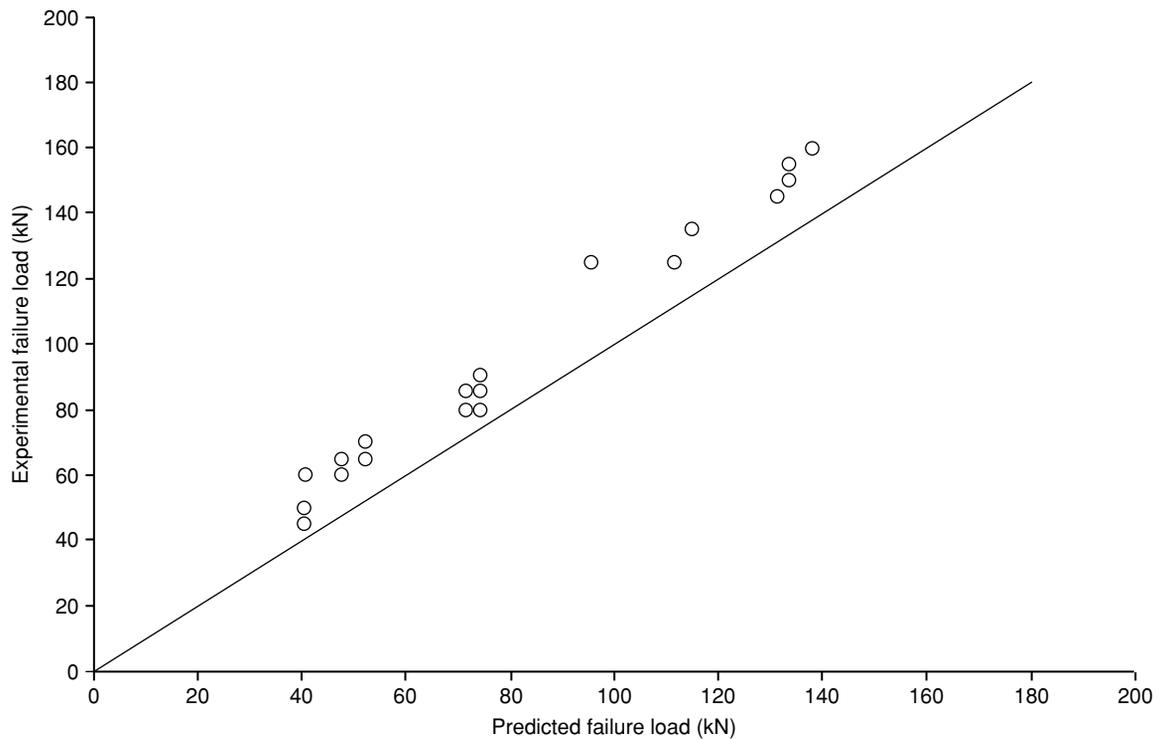


Figure 9. Comparison of predicted failure load with the experimental result

for most of the specimens. The predicted failure load was compared with the experimental failure load and is shown in Figure 6.

From Figure 6, it is seen that the load carrying capacity predicted matches well for most of the cases and further it is seen that majority of experimental results are greater than the predicted failure load.

## 8. EXPERIMENTAL PROGRAMME

In order to validate the equations developed; nineteen simply supported deep beams of size 700 mm  $\times$  350 mm  $\times$  60 mm were cast. Out of this, four beams were tested under UDL, seven beams under two point load and eight beams under central point load. The area of steel was provided such that some beams will fail in flexure and the remaining under shear. The details of the specimen are given in Table 3. The reinforcement details of the beam are shown in Figure 7. Crack control reinforcement as per ACI 318-08 (2008) was provided for all beams. Bearing plates of dimension 60 mm  $\times$  60 mm  $\times$  5 mm were used at supports and at loading points. Effective cover of 25 mm for main tension steel was provided for all beams. Some typical failed specimens are shown in Figure 8. The mode of failure of the beam was identified by comparing the different possible failure patterns of deep beams given in Varghese and Krishnamoorthy (1966).

## 9. DISCUSSION OF TEST RESULTS

The predicted failure load was compared with the experimental failure load and is shown in Figure 9. From Figure 9, it is seen that the predicted values of failure load compares reasonably well with the experimental results. Further, all the predicted capacities are less than the test results. Hence it may be concluded that STM is based on the lower bound theorem of plasticity. From Table 3, it is seen that, the mode of failure predicted matches with the failure mode obtained from experiment for many of the specimens.

## 10. CONCLUSIONS

The present investigation focussed on the analysis and design of simply supported deep beams using strut and tie models. Equations to find the area of main steel required for a deep beam to have a balanced type of failure have been derived. By comparing this with the area of main steel provided in a deep beam, the failure mode can be predicted. Further, equations to predict the ultimate capacity of deep beams failing in different failure modes have been developed. These equations can be used for deep beams subject to any type of loading. The equations developed are validated by comparing with the experimental results. The simplicity of STM and the resulting equations makes it suitable for practical and code implementation.

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## NOTATION

$A_{st}$	area of tension steel reinforcement
$A_{st,b}$	area of main tension steel for balanced type of failure
$b$	width of beam
$C$	compressive force in strut
$C_P$	compressive force in a prismatic strut
$D$	depth of beam
$d$	effective depth of the beam
$e$	effective cover
$f_{cd}$	design compressive strength of concrete
$f_{ck}$	characteristic compressive strength of concrete
$f_{cn}$	permissible stress in nodes
$f_{cs}$	permissible stress in struts
$f_y$	yield stress of steel
$f'_c$	compressive strength of concrete cylinder
$h$	height of truss
$k_i$	shear span coefficient
$L$	effective span
$L_B$	length of bearing plate
$M_U$	factored moment
$P$	capacity of the deep beam
$p_t$	percentage of tensile steel
$R$	support reaction
$T$	tensile force
$w_p$	width of prismatic strut
$w_s$	width of bottle shaped strut
$w_t$	width of tie
$W_U$	factored load
$Z$	lever arm
$\theta$	strut inclination
$\gamma_c$	partial (material) safety factor for concrete
$\gamma_s$	partial (material) safety factor for steel
$\beta_s$	stress reduction factor for strut
$\beta_n$	stress reduction factor for node